

Paper: BSM 3-Algebra-III and Differential equations-I
(SSC 540)

Programme

B.Sc

Subject

Mathematics

Semester

III

University

Kuvempu University

Session

39

Differential Equations of First order and First Degree

Differential equations reducible to
Homogeneous Differential Equations

Recap of previous class

- Solution of DE by separation of variables
- Solution of homogeneous differential equation

Objectives

- Problems to solve differential equations reducible to homogeneous form

Session Outcome:

- able to solve differential equations reducible to homogeneous differential equations

Prerequisites :

- Standard formulae of differentiation
- Standard formulae of integration
- Solution of DE in variable separable form
- Solution of Homogeneous differential equations.

Differential equations reducible to homogeneous form :

The differential equation of the form $\frac{dy}{dx} = \frac{ax+by+c}{a_1x+b_1y+c_1}$
when $\frac{a}{a_1} \neq \frac{b}{b_1}$ can be reduced to homogeneous form

Method of solution:

Consider $\frac{dy}{dx} = \frac{ax+by+c}{a_1x+b_1y+c_1}$ where $\frac{a}{a_1} \neq \frac{b}{b_1}$

Put $x = X + h$ & $y = Y + k$

Find h and K so that $ah + bk + c = 0$

$$\& \quad a_1h + b_1k + c_1 = 0$$

The equation will reduce to homogeneous form $\frac{dY}{dX} = \frac{AX+BY}{A_1X+B_1Y}$

Problems :

1. Solve $\frac{dy}{dx} = \frac{y+x-2}{y-x-4}$

2. Solve $\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$

3. Solve $\frac{dy}{dx} = \frac{y-x+1}{y+x-5}$

4) Solve $(2x + 3y - 5) \frac{dy}{dx} = (3x + 2y - 5)$

5) Solve $\frac{dy}{dx} = \frac{2x - y + 1}{x + 2y - 3}$

Session Summary:

To solve differential equation of the form

$$\frac{dy}{dx} = \frac{ax+by+c}{a_1x+b_1y+c_1} \quad \text{where } \frac{a}{a_1} \neq \frac{b}{b_1}$$

- Put $x = X + h$ & $y = Y + k$
- Find h and K so that $ah + bk + c = 0$
& $a_1h + b_1k + c_1 = 0$
- The equation will reduce to homogeneous form $\frac{dY}{dX} = \frac{AX+BY}{A_1X+B_1Y}$
- This can be solved by putting $Y = vX$

Question :

1) Solve $\frac{dy}{dx} = \frac{1+x+3y}{1-2x-y}$

Ans : $\frac{1}{2} \log(y^2 + 5xy + x^2) + \frac{1}{\sqrt{21}} \log \left[\frac{2y + (5 - \sqrt{21})x}{2y + (5 + \sqrt{21})x} \right] = \log c$

Question :

2) Solve $(x + 2y - 2)dx + (2x - y + 3)dy = 0$

Ans : $x^2 + 4xy - y^2 - 4x + 6y = c$

Question :

$$3) \text{ Solve } (2x + 3y - 5) \frac{dy}{dx} + (3x + 2y - 5) = 0$$

$$\text{Ans : } 3x^2 + 4xy + 3y^2 - 10x - 10y = c$$

Question :

$$4) \text{ Solve } (6x + 2y - 10) \left(\frac{dy}{dx} \right) - 2x - 9y + 20 = 0$$

$$\text{Ans : } (y - 2x)^2 = c(x + 2y - 5)$$

References

- Manjunath, B. V. and Nandeeshkumar(2018).
A textbook of B.Sc Mathematics(First).College book house, Bangalore.
- Raisinghania, M. D.(2014).Ordinary and partial differential equations
(Sixth). S. Chand,New Delhi.