

Paper: BSM 3-Algebra-III and Differential equations-I  
(SSC 540)

**Programme**

**B.Sc**

Subject

Mathematics

Semester

III

University

Kuvempu University

Session

37

# Differential Equations of First order and First Degree

## Homogeneous Differential Equations (HDE)

## Recap of previous class

- Solution of DE by separation of variables  
i.e the solution of the equation of the form

$$Mdx + Ndy = 0$$

where M is a function of x alone and N is a function of y alone is

$$\int M dx + \int N dy = c$$

## Learning Objectives:

- 1) Homogeneous differential equation
- 2) Method of solution of HDE

## Session Outcome :

- Student will be able to solve homogeneous differential equations of first order and first degree.

## Prerequisites :

- Standard formulae of differentiation
- Standard formulae of integration
- Solution of DE in variable separable form

Main Content:

## Homogeneous differential equations

- Homogeneous function :

A function  $f(x, y)$  is said to be homogeneous function of degree  $n$  if

$$f(x, y) = x^n g\left(\frac{y}{x}\right).$$

- Homogeneous Differential Equation :

A D.E of the form  $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$  where  $f(x, y)$  &  $g(x, y)$  are homogeneous functions of same degree in  $x$  &  $y$ .

## Method of solving a homogeneous differential equation :

1) Put  $y = vx$ , then  $\frac{dy}{dx} = v + x \frac{dv}{dx}$ .

2) Separate the variables  $v$  and  $x$ , and integrate.

3) Replace  $v$  by  $\frac{y}{x}$  to get the required solution.

• Also we can put  $x = vy$  ,  $\frac{dx}{dy} = v + y \frac{dv}{dy}$  & replace  $v$  by  $\frac{x}{y}$

.



Problems:

1. Solve  $(x^2 + y^2)dx + 2xy dy = 0$
2. Solve  $(x^2 + 2y^2)dx - xy dy = 0$
3. Solve  $x(y - x)dy = y(x + y)dx$

## Session Summary:

To solve first order first degree homogeneous differential equation

$$\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$$

- put  $y = vx$  &  $\frac{dy}{dx} = v + x \frac{dv}{dx}$  .
- Solve the resulting DE by method of separation of variables.

MCQ :

1. Which of the following is a homogeneous differential equation

A.  $(x^2 - y)dx + (2x^2y)dy = 0$

B.  $(x^3 + 2y^2)dx - 2xydy = 0$

C.  $(x^3 + y^3)dx = (x^2y + xy^2)dy$

D. None of the above

ANS : C

MCQ :

2. Which of the following is not a homogeneous differential equation

A.  $(x^2 - y)dx + (2x^2y)dy = 0$

B.  $(x^3 + 2y^3)dx - 2xy^2dy = 0$

C.  $(x^3 + y^3)dx = (x^2y + xy^2)dy$

D. None of the above

Ans : A

MCQ :

3. Solution of  $(x^2 - y^2)dx + xy dy = 0$  is

A.  $\sqrt{x}(x^2 - 2y^2) = c$

B.  $(x^2 - 2y^2) = c$

C.  $x(x^2 - 2y^2) = c$

D. None of these

Ans : A

MCQ:

4. Solution of  $(x^2 - y^2)dy = 2xy dx$  is

- A.  $y = c(x^2 - y^2)$
- B.  $y = c(x^2 + y^2)$
- C.  $y = c(x^2 - xy)$
- D. None of these

Ans: B.  $y = c(x^2 + y^2)$

MCQ:

5. Solution of  $\frac{dy}{dx} + \frac{x-2y}{2x-y} = 0$

A.  $(x - y)^2 = c(x + y)$

B.  $(x - y)^2 = c(x - y)$

C.  $(x + y)^2 = c(x - y)$

D. None of these

Ans: C.  $(x + y)^2 = c(x - y)$

## References:

- Manjunath, B. V. and Nandeeshkumar(2018). A textbook of B.Sc Mathematics(First).College book house, Bangalore.
- Raisinghania, M. D.(2014).Ordinary and partial differential equations (Sixth). S. Chand,New Delhi.