BSM 6-Line and Multiple integrals and Laplace Transforms

Programme	**B.Sc**
Subject	Mathematics
Semester	V
University	Kuvempu University
Session	5

Double integration

Definition of double integrals and Examples

Recap of previous class

- Definition of line integral and basic properties

Objectives:

1. Definition of double integrals
2. Examples on double integrals

Session outcomes

- To able to understand the double integration
- To able to understand properties of double integration
- To able to understand problems

Prerequisites

- Standard formulae of integration

Double integration

The definite integral $\int_a^b f(x)dx$ is defined as the limit of sum $f(x_1)\delta x_1 + f(x_2)\delta x_2 + f(x_3) + \cdots + f(x_n)\delta x_n$ Where n$\rightarrow \infty$ and each of the lengths $\delta x 1$,$\delta x 2$,…………..$\delta x n$ tends to zero. A double integral is its counterpart in two dimensions.

Consider a function $f(x, y)$ of the independent variable x, y defined at each point in the finite region A of the xy-plane.

Divided A into n elementary areas $\delta A_1, \delta A_2, \delta A_3 \dots \dots \dots . . \delta A_r$.

Let (x_r, y_r) be any point with in the r^{th} elementary area δAr. Consider the sum $f(x_1, y_1)\delta A_1 + f(x_2, y_2)\delta A_2 + f(x_3, y_3)\delta A_3 + \cdots + f(x_n, y_n)\delta A_n$

i.e$\sum_{r=1}^{n} f(x_r, y_r)\delta A_{r.}$

The limit of this sum, if it exists, as the number of sub-divisions increases indefinitely and area of each sub-division decreases to zero, is defined as the double integral of $f(x, y)$ over the region A and is written as

$\iint_A f(x, y)dA$

Or $\iint_A f(x, y)dxdy$.

Thus $\iint_A f(x, y)dA = \lim_{\substack{n\to\infty \\ \delta A \to 0}} \sum_{r=1}^{n} f\left(x_{r,} y_r\right)\delta A_r \ldots . . \delta A_r \ldots \ldots (1)$

If the region A is bounded by the curves $x = x_1, x = x_2$ and $y = y_1, y = y_2$ then $\iint_A f(x, y)dxdy$ =$\int_{y1}^{y2} \int_{x1}^{x2} f(x, y)dxdy$.

Evaluation of Double integrals

a) If $x_1, x_2, y_{1,} y_2$ are constants, then the order of integration is immaterial, provided the limits of integration are changed accordingly.

Thus $\iint_A f(\mathrm{x,y})\mathrm{dxdy} = \int_{y1}^{y2}\int_{x1}^{x2} f(x,y)dxdy = \int_{x1}^{x2}\int_{y1}^{y2} f(x,y)dydx$

b) If $x_{1,}x_2$ are functions of y, let $x_1 = \varphi_1(y)$ and $x_2 = \varphi_2(y)$ and $y_{1,}y_2$ are constants, then $f(x,y)$ is first integrated with respect to x keeping y fixed, with in the limits x_1, x_2 and the resulting expression is integrated w.r.to y between the limits y_1 and y_2i.e$\iint_A f(x,y)dxdy = \int_{y1}^{y2}\left[\int_{x1}^{x2} f(x,y)dx\right]dy$.

c) If y_1, y_2 are functions of x, let $y_1 = \varphi_1(x)$ and $y_2 = \varphi_2(x)$ and x_1, x_2 are constants, then $f(x,y)$ is first integrated w.r.to y keeping x fixed, with in the limits y_1, y_2 and the resulting expression is integrated w.r.to x between the limits x_1 and x_2 i.e $\iint_A f(x,y)dxdy = \int_{x1}^{x2}\left[\int_{y1}^{y2} f(x,y)dy\right]dx$.

Problems:

1) Evaluate $\int_0^1 \int_0^1 (x^2 + y^2) dx dy$

2) Evaluate $\int_0^3 \int_0^1 xy(x + y) dy dx$

3) Evaluate $\int_0^a \int_0^b xy(x - y) dy dx$

4) Evaluate $\int_1^2 \int_2^5 x dx dy$

Session Summary:

- To solve double integration, which is useful in finding certain areas.
- The knowledge of integration is a must before tackling double integrals

MCQ :

1) The solution of $\int_0^1 \int_0^1 \frac{dxdy}{\sqrt{(1-x^2)(1-y^2)}}$

A. $\frac{\pi^2}{4}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{6}$

D. None of the above

Ans : A

MCQ :

2) The solution of $\int_0^2 \int_0^{x^2} x(x^2 + y^2)dydx$

A. $\frac{64}{2}$
B. $\frac{64}{3}$
C. $\frac{1}{2}$
D. 0

Ans : B

MCQ :

3) The solution of $\int_0^4 \int_0^{\sqrt{y}} xydxdy$

$A.\ \frac{32}{3}$

$B.\ \frac{231}{2}$

C. 250

D. None of these

Ans : A

MCQ:

4) $\int_0^2 \int_{-y}^{\sqrt{y}} (1 + x + y)\, dxdy$

A. $\frac{13}{3} + \left(\frac{52}{15}\right)\sqrt{2}$
B. $\frac{13}{2}$
C. $\frac{13}{3} + \left(\frac{52}{5}\right)\sqrt{3}$
D. None of the above

Ans: A

References:

- Manjunath, B. V. and Nandeeshkumar(2018). A textbook of B.Sc Mathematics. College book house, Bangalore.

- Ranganath G. K (2012). A textbook of B.Sc Mathematics (Sixth). S. Chand, New Delhi.