I Semester B.Sc/B.A. Examination, Nov/Dec 2005

(Semester Scheme)

MATHEMATICS (Paper-I)

Time: 3 Hours Max.Marks:90

Instructions: 1) Answer all questions.

2) Answer should be written completely either in English or in Kannada.

1. Answer any fifteen of the following: (2x15=30)

1) Write the negation of V x [P (x) $\rightarrow$ q(x)].

2) Find T [P (x)], if P(x): x^2-3x+2=0, the replacement set is z.

3) Define an equivalence relation.

4) If f: Q$\rightarrow$Q is defined by f(x)=3x+1, Vx€Q, then show that f is onto.

5) Find the nth derivative of $C^{x/2}$, Sin 2x.

6) Find the nth derivative of $\sin^3 2x$.

7) If z=xy, then find $\partial^2 z/\partial x\, \partial y$

8) If y^2=4ax, find dy/dx using partial differentiation.

9) If u=x^3-2x^2y+3xy^2+y3, prove that x $\partial u/\partial x$+y $\partial u/\partial y$=3u.

10 If u=2x-3y_∞, v=5x+4y, show that ∂ (u,v)/ ∂ (x,y)=23.

11) Evaluate $\int dx/ (1+x^2)^{7/2}$

12) Evaluate $\int^{\pi 2}\sin^4 x \cos^3 x\, dx$.

13) Find the direction cosines of the joining (2,-3, 6) and (3, -1,-6).

14) Show that the line x-1/-1= y+1/1=z/1 lies on the plane x-y+2z=2.

15) Find the equation of a plane passing through (2,3, 4) and perpendicular to the vector ^i+^j+^k.

16) Express the equation of the plane 3x-y+6z=9 in the normal form.

17) Find the distance between the parallel planes 2x-y+3z=-4 and 4x-2y+6z-6=0.

18) Find the angle between the plane 2x+y+2z=5 and the line x-1/2= y+1/-1 = z-1/2.

19) Find the equation of a sphere concentric with the sphere 2x^2 +2y^2+2z^2-4x+6y-8z+1=0, passing through (2,1,-3)

20) Find the equation of cone whose vertex is at the origin, the axis is x/2=y/1=z/3 and the

semi vertical angle is 30˚.

II. Answer any two of the following: (5x2=10)

1) With the usual notation, prove that:

T[P (x) → q(x)]= [T[P (x)] UT [q(x)].

2) a) Symbolise and negate ‘Some integers are perfect squares or all integers are rational numbers’

b) Give the direct proof of the statement:

‘If x+y is even, then x any y are both odd or both even’ where x and y are integers.

3) Prove that any partition P of a non empty set A determines an equivalence relation. On A.

4) If f:R→R; g:R →R are defined by f(x)=2x+1 and g(x)=5-3x, verify $(gof)^{-1}=f^{-1}$ o g^{-1}.

III. Answer any three of the following: (5x3=15)

1) Find the nth derivative of 3x+2/ x^3+x^2
2) If y=cos(m $\sin^{-1}$ x), show that $\frac{yn+1}{7n} = \frac{4n^2-m^2}{4n+2}$ at x=0.
3) State and prove Leibniz’s theorem for nth derivative of product of two functions.
4) If u= x/y+z + y/ z+x + z/x+y, show that x ∂u/∂x+ y ∂u/∂y+z ∂u/∂z=0.
5) If u=2xy, $v=x^2-y^2$, x=r cos ø, y=r sin ø. prove that ∂ (u, v)/ ∂ (r, 0) = -4r3.

IV. Answer any two of the following: (5x2=10)

1) Obtain the reduction formula for $\int^{\pi/2} \sin^m x \cos^n x\, dx$.
2) Using Leibniz’s rule for differentiations under the integral sign, evaluate $\int \frac{x^a-1}{\log x} dx$, where a is a parameter.
3) Evaluate $\int^{\sqrt{2}} \frac{x3\, dx}{\sqrt{x}^{2-1}}$

V. Answer any three of the following: (5x3=15)

1. Find the volume of the tetrahedron formed by the pints. (1, 1, 3), (4, 3, 2,) (5, 2, 7) and (6, 4, 8).
2. If a line makes angles ∝ β ϒu,B,Y and __ with four diagonals of a cube, show that $\cos^2$ u$+\cos^2$ β$+\cos^2$ ϒ$+\cos^2$ *S*=4/3.
3. Prove that the lines (x-5)/4 = (y-7)/4 = (z-3)/-5 and (x-8)/7 = (y-4)/1 = (z-5)/3 are coplanar. Find the equation of the plane containing them.
4. Find the length and foot of the perpendicular drawn from (1, 1, 2) to the plane 2x-2y+4z+5=0.
5. Find the shortest distance between the lines (x-8)/3 = (y+٥)/-16 = (z-10)/7 and (x-15)/3 = (y-29)/8 = (z-5)/-5.

VI. Answer any three of the following: (5x2=10)

1. Find the equation of a sphere passing through the points (1, 0, 0), (0, 1, 0), (0, 0, 1), and (2,-1, 1). Find its centre and radius.

2. Derive the equation of a right circular cone in the standard form $x^2+y^2=z^2 \tan^2 \acute{\alpha}$.
3. Find the equation of right circular cylinder of radius 3 units, whose axis passes through the point (1, 2, 3) and has direction rations (2,-3, 6).

SECOND SEMESTER B.Sc./B.A. EXAMINATION.

APRIL/MAY 2005

(Semester Scheme)

MATHEMATICS (PAPER-II)

Time: 3 Hours Max. Marks: 90

Instructions: 1. Answer all questions.

2. Answers should be written completely either in English or in Kannada.

(2x15=30)

I. Answer any fifteen of the following:

1. If 0 is an eigen value of square matrix A, then prove that A is singular.
2. For what value of x is the rank of the matrix a equal to 3 given.
   A= 2 4 2

   3 1 2

   1 0 x

3. 3. Find the value of k such that the following system of equations has non-trivial solutions.(k-1)x+ (3k+1)y+2kz=0 (k-1)x+ (4k+2)y+(k+3_z=0 2x+(3k+1)y+3 (k-1)z=0.
4. Find the eigen value of the metric A= a h g
   0 b 0

   0 c c

5. For the matrix A= 3 1
   -1 2 the characteristic equation is $\lambda^2-\lambda$ -5=0. Using it, find A-1.

6. For an equiangular spiral r=a $e^{\theta \cot}$a show that the tangent at every point is inclined at a constant angle with the radius vector.
7. Find the pedal equation of the carver r=aθ.
8. Show that for the curve rθ=a the polar sub tangent is a constant.
9. In the curve $pa^n=r^{n+1}$, show that the radius of curvature varies in-versely as the (n-1)th power of the radius vector.
10. Show that the origin is conjugate point of the curve $x^2+3y^2+x^3y=0$.
11. Find the envelope of the family of lines y=mx+a/m, where m is a parameter.
12. Find the asymptotes (if any) of the curve $x^3y^2+x^2 y^3=x^3+y^3$ parallel to the y-axis.
13. Prove that $y=e^x$ is every where concave upwards.
14. Find the length of the arc of the semi-cubical parbola $ay^2=x^3$ from the vertex to the point (a,a)
15. Find the whole area of the circle r=2a cos θ.
16. Solve: dy/dx+y/x=1/x
17. Solve: dy/dx+y=sin x,

18. Find the integrating factor of the equation x dy-y dx+2x^3 dx=0.
19. Solve: p2+p(x+y)+xy=0 where p=dy/dx.
20. Find the singular solution of y=xp+p2.

II. Answer any three questions: (3x5=15)

1. Find the rank of the matrix a by reducing to the normal form given:A= 1 -1 -2 -4
   2 3 -1 -1
   3 1 3 -2
   3 -3 4 6 3 0 -7
2. If A= 2 -3 4 determine two non-singular matrices P and Q such that PAQ=I, Hence find A-1.
   0 -1 1
3. For what values of λ and μ the equations:x+y+z=6 x+6y+3z=10 x+2y+ λz= μ have (1) no solution (2) a unique solution (3) infinite number of solutions.
   1. Find the eigen values and eigen sectors of the matrix: A= 1 2 3
      0 -4 2
      0 0 7
   2. State and prove the Cayley-Hamilton theorem.

III. Answer any three questions: (2x5=10)

1. Find the angle of intersection of the parabolas r= a/1+cos0 and r=b/1-cos0
2. Show that the p-r equation of the curves x=a $\cos^3$ 0 and y=a $\sin^3$ 0 is $r^2=a^2-3p^2$.
3. For the curve x=x(t)y=y(t) show that the radius of curvature. P=$\frac{[x^2+y^2]^{3/2}}{xÿ-ýx}$
4. Prove that the evolute of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}$= 1 is $(ax)^{2/3}+(by)^{2/3}=(a^2-b^2)^{2/3}$

IV. Answer any three questions: (2x5=10)

1. Find the points of inflexion on the curve x=10g[y/x)
2. Determine the position and nature of the double points on the curve u\y(y-6)=$x^2(x-2)^3$-9.
3. Find all the asymptotes of the curve 4x^2(y-x)+y (y-2) (x-y)=4x+4y-7.
4. Trace the curve y^2(a-x)=x^2(a=x)

V. Answer any three questions: (2x5=10)

1. Find the perimeter of the asteroid $x^{2/3}+^{y2/3}=a^{2/3}$.
2. Find the surface area generated by revolving an arch of the cycloid x=a(0-sin 0) y=a (1-cos 0) a bout the x-axis.
3. Find the volume of the solid generated by revolving the cardioid r=a (1+cos 0) about the initial line.

VI. Answer any three questions: (3x5=15)

1. Solve: dy/dx +y cos x=y^n sin 2x

2. Solve: dy/dx= x+2y-3/ 2x+y-3.

3. Solve: 2y dx+(2x log x-xy) dy=0.

4. Show that the family of parabolas y^2=4a (x+a) are self orthogomal.

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III SEMESTER BSc. MATHEMATICS

MODEL QUESTION PAPER-III

Time: 3 Hours Max. Marks: 90

I. Answer any fifteen of the following 15x2=30

1. Show that O(a)=O(xax-1) in any group G.
2. Define center of a group.
3. Prove that a cyclic group is abelian.
4. How many elements of the cyclic group of order 6 can be used as generator of the group?
5. Let H be a subgroup of group G.Define K={xϵG:xH=Hx}Prove that K is a subgroup of G.
2. Find the index of H={0,4} in G= (z_8, $+_8$).
3. Define convergence of sequence.
4. Find the limit of the sequence √2, √2 √2, √2√2√2..............
5. Verify Cauchy's criterion for the sequence {n/n+1}
6. Show that a series of positive terms either converges or diverges.
7. Show that 1/1.2 + 1/2.3 + 1/3.4+.......... is convergent.
8. State Raabe's test for convergence.
9. discuss the absolute convergence of 1-x^2 + ∟2 x4 4! - x6 6!+......When x2=4

10. If a and b belongs to positive reals show that a-b a+b +1/3 $[a-b\ a+b]^3$ +1/5 $[a-b/a+b]^5$+.....
11. Name the type of discontinuity of (x)={3x+1, x>1
    2s-1, x≤1}

12. State Rolle's theorem.
13. Verify Cauchy's Mean Value theorem for f(x)=log x and g(x)=1/x in {1, e]
14. Evaluate lim $(cotx)^{\sin 2x}$ x→ 0
15. Find the Fourier coefficient a0 in the function foo = x, o ≤x <π
    2π -x, π≤x≤2 π

16. Find the half range sine series of (x)=x over the interval (0, π).

II. Answer any three of the following 3x5=15

1. Prove that in a cyclic group (‹a) or order d, a (k<d) is also a generator iff (k,d)=1.
2. Show that {[1 0 [-1 0 [-1 1 [0 -1 [1 -1 form a cyclic group w.r.t. matrix multiplication.
   0 1], 0 -1], -1 0], 1 -1], 0 0]}

3. Prove that if H is a subgroup of g then there exist a one-to-one correspondence between any two right cosets of H in G.
4. Find all the distinct cosets of the subgroup H= {1, 3,9} of a grouop G= {1,2,.....12} w.r.t multiplication mod 13.
5. If a is any integer p is a prime number then prove that a^p=(amod p).

III. Answer any two of the following 2x5=10

1. Discuss the behavior of the sequence $\{(1+ 1/n)\}^n$
2. If an=3n-4/4n+3 and |an - ¾/< 1/100, n> m find m using the definition of the limit.

3. Discuss the convergence of the following sequences whose nth term are (i) $(n^2-1)1/8$ - $(n+1)¼$ (ii) [log(n+1)-log n]/tan (1/n)

IV. Answer any fifteen of the following 2x5=10

1. State and prove P-series test for convergence.
2. Discuss the convergence of the series 1.2/456 + 3.4/6.7.8 + 5.6/8.9.10+..........
3. Discuss the convergence of the series x^2 2√1+x^3 √3√2 + x^4/4√3+........
4. Show that ∑ (-1)n / (n+1)p is absolutely convergent if p›1 and conditionally convergent if p z 1
5. Sum the series $\sum(n+1)^3/n!]$ xn

V. Answer any two of the following 2x5=10

1. If $\lim_{x\to a} f(x)=L$, $\lim_{x\to a} g(x)=m$ prove that $\lim_{x\to a}$ [(x)+g (x)=1+m
2. State and prove Lagrange's Mean Value theorem.
3. Obtain Maclaurin's expansion for log (1+sin x)
4. Find the yalues of a, b, c such that $\lim_{x\to a}$ x(2+a cosx)-b sin x+c / x5 xo5 = 1/15

VI. Answer any two of the following 2x5=10

1. Expand $f(x)=x^2$ as a Fourier series in the interval $(-)_{(\lambda-\pi,-\pi)}$ and hence Show that $1/1^2+1/2^2+1/3^2+.....\ \pi^{2/6}$
2. Find the cosine series of the function (x)= π-x in 0< x<_ π.
3. Find the half-range sine series for the function (x)=2x-1 in the interval (0.1)

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BSc, IV SEMESTER MATHEMATICS

MODEL QUESTION PAPER-1

Time: 3 Hours Max. Marks: 90

I. Answer any fifteen of the following 2x15=30

1. Prove that every subgroup of an abelian group is normal.
2. Prove that intersection of two normal subgroups of a group is also a normal subgroup.
3. The center Z of a group G is a normal subgroup of G.
4. Define a homomorphism of groups.
5. If $G=\{x+y\sqrt{2}|x,y\epsilon Q\}$ and $f:G\rightarrow G$ is defined by $f(x+y\sqrt{2})=x-y\sqrt{2}$, show that f is a homomorphism and find its kernel.
6. Show that $f(x,y)= \sqrt{|xy|}$ is not differentiable at (0,0).
7. Show that $f(x,y)=\tan^{-1}[y\ x)$ at (1, 1) has limit.
8. Prove that there is a minimum value at (0,0) for the fuctions x^3+y^3-3xy.
9. Show that ${}_{o}\int^{\pi/2}\cos^{10} 0d0=\frac{1}{2}\beta\left(\frac{11}{2}, \frac{1}{2}\right)$
10. Prove that $\Gamma(n+1)=n!$
11. Prove that ${}_{o}\int^{\infty 2} \sqrt{x}e^{-x2}\, dx=\frac{1}{2}|^{\rightarrow 3/4}$
12. Find the particular integral of $y^{11}-2y`+4y=e^x \cos x$.
13. Show that $x(2x+3)y^{11}+3(2x+1)y^1+2y=(x+1)e^x$ is exact.
14. Verify the integrability condition for yzlogzdx-zxlog zdy+xydz=0.
15. Reduce $x^2\ y^{11}-2xy^1 1+3y=x$ to a differential equation with constant coefficients.
16. Find $\llcorner[\sin^{3t}]$.
17. Find $\{\llcorner^{-1}\ (s+2)\ (s-1)\}$
18. Define convolution theorem for the functions f(t) and g(t).
19. Find all basic solutions of the system of equations: 3x+2y+z=22, x++y+2z+9.
20. Solve graphically $x+y\leq 3$, $x-y \geq -3$, $Y \geq 0$, $x\geq -1$, $x\leq 2$.

II. Answer any two of the following 2x5=10

1. Prove that a subgroup H of a group G is a normal subgroup of G if and only if the product of two right coses of H in G is also a right coset of H in G.
2. Prove that the product of two normal subgroups of a group is a normal subgroup of the group.
3. If G and G^1 are groups and $F:G\rightarrow G^1$ is a homomorphism with kernel K, prove that K is a normal subgroup of G.
4. If G=(Z6, +6), G^1=(Z2, +2) and the function f: $G\rightarrow G^1$ is defined by f(x)=r where r is the remainder obtained by dividing x by 2, then verify whether f is homomorphism. If so, find its kernel. Is f an isomorphism?

III. Answer any three of the following 3x5=15

1. State and prove Taylor's theorem for a function of two variables.
2. Find Maclaurin's expansion of log (1+x-y).
3. Find the stationary points of the function $f(x,y)=x^3y^2(12-x-y)$ satisfying the condition x>0 and examine their nature.
4. Show that ${}_{o}\int^{\infty}\frac{x^4(1+x^5)}{(1+x)^{15}}\, dx = \frac{1}{5005}$

OR

If n is a positive integer, prove that $\Gamma^{1}(m+1/2)= \frac{1.3.5......(2n-1)\sqrt{\pi}}{2^{n}}$

5. Show that $_{o}\int^{\pi/2} \sqrt{sin0}d0.\ 1\ sin0\ do\ +\ _{o}\int^{\pi/2} \frac{1\ \ 2^{n}}{\sqrt{sino}\quad d0}$

OR

Evaluate $_{o}\int^{\infty} \frac{dx}{01+x^{4}}$ ◦

IV. Answer any three of the following 3x5=15

1. Solve $y^{11\ 1}-2y^{11}+4y=e^{x}$ cosx.
2. Solve $x^{3}y^{111}+2x^{2}y+2y^{11}+10$ (x+1/x).
3. Solve $\frac{d^{2}y}{dx^{2}}$ -$(1+4e^{x})$ $\frac{dy}{dx}$ + $3e^{2x}y=e^{2}(x+e^{x})$ using changing the independent variable method.

4. Solve dx/dt+3x - y, dy/dt=x+y
5. Solve dx $x^{2}+y^{2}+yz$ = dy/$x^{2}+y^{2}$-zx = dz/z(x+y)

V. Answer any two of the following 2x5=10

1. Find (i) ∟{2sin st sin 5t/t (ii) ∟-1log s^{2}+1 s(s+1)}
2. Verify convolution theorem for the functions (t)=et and g(t)=cost.
3. Solve $y^{11}+2y^{1}$=10sin tsy given $y(0)^{zo}$, y(0)=1 using Laplace transform method.

VI. Answer any two of the following: 2x5=10

1. Find all the basic feasible solutions to the LPP:

Maximize: z=2x+3y+4z+7t

Subject to the constrains: 2x+3y-z+4t=8, x-2y+6z-7t=-3 x,y,z,t >0.

2. A quality engineer wants to determine the quantity produced per month of products A and B.

Source	Product A	Product B	Available month
Material Working hours Assembly man hours	60 8 3	120 5 4	12,000 600 500
Sale price	Rs. 30	Rs. 40	-

Find the product mix that give maximum profit by graphical method.

3. Using Simplex and method to maximize $f=5x+y+4z$ subject to $x+z<8$, $y+z<3$ $x+y+z<5$.

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MATHEMATICS MODEL PAPER-3; B.Sc., FIFTH SEMESTRE

PAPER-V

Time: 3 hours Max Marks:90

I. Answer any fifteen Questions: 2x15=30

1. In a ring (R,+,.) Prove that a.0=0 a=0, VaєR, where 0 is identity in R.
2. Give an example of a non commutative ring, without unity and with zero divisors.
3. show that Z is not an ideal of the ring (Q, +,.)
4. If the additive group of a ring R is cyclic then prove that R is Commutative.
5. With an example, show that union of two sub rings of ring need not be a sub ring.
6. Show that (z,+,.) and (2z,+,.) are not isomorphic defined by f(x)=2xVxєz
7. If→ (t)=â coswt+b sin wt, show that $\frac{d^2\vec{r}}{dt2}$ =-w^2→r (a and b are constant vectors).
8. Show that the necessary and sufficient condition for the vector a(t) to have constant magnitude is →a. da →/d t =0.
9. For the space curve →r=ti+t^2j+2/3t^3k, find the unit tangent vector at t=1.
10. Write the Serret-F rennet formula for the space curve →r= →r(s).
11. Find the Cartesian co ordinates of the point whose cylindrical coordinates are (3,3,5) $\frac{\pi}{}$
12. Find the unit normal to the surface 4z=x^2-y^2 at (3, 1, 2)
13. Prove that div(curl →F)=0
14. Show that →F=(sin y+z)i+(xcosy-z)j+(x-y)k is irrigational
15. If a is a constant vector, show that curl (→rx→a)=-2a→
16. Find a, so that →F=y(ax^2+z)i+x(y^2-z^2)j+2xy(z-xy)k is solenoidal.
17. Using Rodrigue's formula obtain expressions for Po(x) and P_1x)
18. Evaluate $\int_{-1}^{1} x^3 P_4(x)dx$
19. Starting from the expressions of J1/2(x)and J-½(x)in the standard form prove that $\int_o^{\pi/2}\sqrt{x}$ $J_{½}(2x)dx=1/\sqrt{\pi}$
20. Using the expansion of e $^{x/2}$(t- 1/t), show that -n(x)=Jn(-x).

II Answer any four of the following 4x5=20

1. Prove that R={0,1,2,3,4,5}is a commutative ring w.r.t. (+mod 6) and xmod 6
2. Show that the intersection of two subrings is subring and give an example to show that the union of two subrings need not be a subring.
3. Prove that an ideal S of the ring of integers (z,+,.) is maximal if and only if S is generated by some prime integer.
4. Define kernel of a ring homomorphism. If f: R›R^1 be a homomorphism or R in to R^1, then show that Ker f is an ideal of R.
5. If f:R-R^1 be a isomorphism of rings R into R', then prove that if R is a field then R' is also a field.
6. Define Quotient ring. If I is an ideal of the ring, R, then show that the quotient R/I is homomorphism image of R with I as its Kernel.

III Answer any three of the following: 3x5=15

1. Derive the expressions for curvature and torsion in terms of the derivatives of rw.r.t parameter u, where r=r(u)is the equation of the curve.

2. For the space curve x=t, $y=t^2$, $z=2/3t^3$, find (i)t and (ii) k at t=1.
3. Find the equations of the tangent plane and normal line to the surface $2z=3x^2+4y^2$ at the point (2, -1,8)
4. Find the constants a and b so that the surface ax2-byz=(a+x)x will be orthogonal to the surface $4x^2 y+z^3=\rightarrow 4$ at (1, -1,2).
5. Express the vector f=zi-2xj+ykin terms of spherical polar coordinates and find fr fo and f

IV. Answer any three of the following: 3x5=15

1. Find the directional derivative of $=x^2yz+4xz^2$ at (1,2-1) in the direction of the vector 2i-j-2k. In what direction the directional derivative is maximum. What is the magnitude of the maximum directional derivative.
2. Show that $\Delta.\frac{\{f(r)r\rightarrow\}}{r}=\frac{1}{r^2}\frac{d}{dr}\{r^2 f(r)\}$ where r =xi+y+zk
3. Show that $F=(x^2-yz)i+(y^2-zx)j+(z2-xy)k$ is irriotional. Find such that $\rightarrow F=\Delta$
4. For any vector field F, prove that curl curl F=Δ (div F)- Δ^2F
5. Derive an expression for curl F in orthogonal curvilinear coordinates.

V. Answer any two of the following (2x5=10)

1. Prove that $x^4-3x^2+x=\frac{8}{35}P_4(x)-\frac{10}{7}P^2(x)+p1(x)-\frac{4}{5}P_o(x)$.
2. Show that $(n+1)P_{n+1}(x)=(2n+1)x\, P_n(x)-n\, P_{(n-1)}(x)$
3. Prove that $J_{1/2}(x)=\frac{\sqrt{2}}{\pi x}\sin x$
4. Show that $J_o(x)=\frac{1}{\pi}\int_o^{\pi}\cos(xsino)do=\frac{1}{\pi}\int_o^{\pi}\cos(xcoso)do$

OR

Prove that $\cos(xsino)=J_0(x)+2\sum_{n-1}^{\infty}J_{2n}(x)\cos 2n0$

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MATHEMATICS MODEL PAPER 3:B.Sc, FIFTH SEMESTER

PAPER-VI

Time: 3 hours Max Marks: 90

I Answer any fifteen questions:

1. Form a partial differential equation by eliminating the arbitrary constants $(x-a)^2+(y-b)^2+z^2=r^2$
2. Solve pq=k
3. Solve p2=qz
4. Find the particular integral of $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x\,\partial y} + \frac{r^2 z}{\partial y^2} = 12xy$
5. Use the method of separation of variable solve $x\frac{\partial u}{\partial x} + \frac{\partial u}{oy} = 0$
6. Evaluate $\Delta e^{3x} \log 4x$
7. Prove that $\frac{\Delta}{\Delta} - \frac{\Delta}{\Delta} = \Delta + \Delta$
8. Estimate the missing term from the table

X	0	1	2	3	4
y	1	3	9	…	81

9. Write the Lagrange's inverse interpolation formula
10. Evaluate $\int \frac{dx}{1+x}$ using Simpson's 1/3 rule
11. If a particle of mass 2 units moves along the space curve defined byr→$(4t^2-t^3)i-5t\,j+(t^4-2)k$, find its kinetic energy at t=1.
12. In a SHM if →f the acceleration, u the velocity at any instant and T is periodic time, show that $f^2T^2+4\pi^2v^2$ is a constant.
13. If man can throw a stone to a distance of 100m. How long it is in the air.
14. Prove that if the time of flight of a bullet over a horizontal range R is T, the inclination of projection to the horizontal is $\tan^{-1}\left(\frac{gT^2}{2R}\right)$
15. If the angular velocity of a point moving in a plane curve be constant about a fixed origin, show that its transverse acceleration varies as its radial velocity.
16. When a particle of mass m moves outside a smooth circle of radius r, mention the equation of motion at any point on it.
17. Find the law of force when the particle describes r=a cos0 under the action of central force.
18. Define Apsidal distance and Apsidal angle.
19. A system consists of masses 1,2,3 and 4 units moving with velocities 8i, 7i, 3k and 2i+3J-*k* respectively. Determine the velocity of the mass centre.
20. Briefly explain the angular momentum of a system of particles.

II. Answer any three of the following: 3x5=15

1. Form the partial differential equation by eliminating the arbitrary function z=y f(x +xg (y

y)

x)

2. Solve $(x^2+y^2)(p^2+q^2)=1$
3. Solve $x(x^2+3y^2)p-y(3x^2+y^2)q=2z(y^2-x^2)$

OR

Solve by Charpit's method $q=(z+px)^2$

4. Solve $(D^2-DD^1)z=\cos x \cos 2y$
5. Reduce the equation to canonical form $x^2(y-1)r-x(y^2-1)s+y(y-1)t+xyp-q=0$
6. Solve by using the method of separation of variables $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=3(x^2+y^2)u$

OR

An insulated rod of length l has its ends A and B maintained at 0˚c and 100˚c respectively until steady state conditions prevail. If B is suddenly reduced to 0˚c and maintained at a, find the temperature at a distance x from A at time t.

III. Answer any three of the following: 3x5=15

1. Show that the nth differences of a polynomial of the nth degree are constant and all higher order differences are zero.

OR

By separation of symbols prove that $4x - \frac{1}{8}\Delta^2 4_{x-1} + \frac{1.3}{8.16}\Delta^4 4_{x-2} - \frac{1.3.5}{8.16.24}\Delta^6 4_{x-3} = 4x=\frac{1}{2} -\frac{1}{2}$

$\Delta^6 4_{x-1/2} +\frac{1}{4}\Delta^2 4_{x-1/2} +\frac{1}{8}\Delta^3 4_{x-1/2}$

2. Evaluate $y=e^2x$ for x=0.05 from the following table

X	0.00	0.10	0.20	0.30	0.40
Y=e2x	1.000	1.2214	1.4918	1.8221	2.255

3. Use Lagrange's formula find a polynomial to the following data and hence find f(2)

X	0	1	3	4
F(x)	-12	0	6	12

4. Find (5) using f(1)=3, f(3)=31, f(6)=223, f(10)=1011 and f(11)=1343
5. Find the value of $51 \int_1 \log_{10} x\, dx$ taking 8 sub-intervals correct to four decimal places by Trapezoidal rule.

IV. Answer any four of the following: 4x5=20

1. A particle of mass 2 moves in a force field $\vec{F}=24t^2 i+(36t-16)j-12tk$. Find (i) the kinetic energy at t=1 and t=2 (ii) work done in moving the particle from t=1 to t=2.

2. If the displacement of a particle moving in a straight line is expressed by the equation x=a cons nt+b sin nt. Show that is describes a SHM. Also find its (i) amplitude (ii) periodic time.
3. A particle is thrown over a triangle from one end of a horizontal base and grazing over the vertex falls on the other end of the base. If A,B be the base angles of the triangle and γ the angle of projection, prove that tan γ =tan A+tan B.
4. The angular elevation of an enemy's position on a hill h feet high is β. Show that in order to shell it the initial velocity of the projectile must not be less than $\sqrt{hg(1+\operatorname{cosec}\beta)}$
5. The velocities of a particle along and perpendicular to the radius vector from a fixed origin are λr^2 and $\mu 0^2$. Show that the equation to the path is $\frac{\lambda}{\theta} = \frac{u}{2r^2} + c$ and components of acceleration are $2\lambda r^2 r^3 - \frac{\mu^2 \theta^4}{r}$ and $2\lambda\mu\,\theta^2 - \frac{\mu^2 \theta^3}{r}$
6. A particle projected along the inner side of a smooth circle of radius a, the velocity at the lower point being u. Show that if $2a\,g<u^2<5ag$ the particle will leave the circle before arriving at the highest point and will describe a parabola whose latus rectum is $\frac{2(u^2-2ag)^3}{27a^2 g^3}$

V. Answer any two of the following: 2x5=10

1. Derive the differential equation of a central orbit in pedal form h^2 p3 (dp/dr)=f
2. A particle describes the cardioid r=a(1+cos0) under a central force to the pole. Find the law of force.
3. A particle moves with a central acceleration $\mu(r+a^4/r3)$ being projected from an apse at a distance 'a' with velocity $2\sqrt{\mu}a$ Prove that is describes the curve $r^2(2+\cos\sqrt{3}\theta) =3a^2$
4. Define mass centre of a system of particles and show that the linear momentum of a system of particles relative to its mass centre is zero.

BANGALORE UNIVERSITY

VI Semester B.Sc.,

MATHEMATICS (Paper VII)

Model Paper-I

Time: 3 hours Max. Marks: 90

I. Answer any fifteen questions: 2x15=30

1. Define vector space over a field.
2. Prove that the subset W={x,y,z) {x=y=z} is subspace of V_3 (R).
3. Prove that the set S={(1,0,0), (0,1,0), (0,0,1)} is linearly independent in V_3 (R).
4. Show that the vectors {(1,2,1), (2,1,0), (1,-1,2)} form a basis of V_3 (R).
5. Prove that T: V_3(R)-V_2(R) defined by T(x,y,z)=(x,y) is a linear transformation.
6. Find the matrix of the linear transformation T: V_3(R)-›V_2(R) defined by T(x,y,z)=(x+y,y+z) w.r.t. standard basis.
7. Define Rank and Nullity of the linear transformation.
8. Evaluate ∫(x+y dx+(y-x) dy along the parabola y2=x from (1,1) to (4,2)
9. Evaluate $_0\int^1 {}_0\int^{\sqrt{1+x2}} \frac{dy\ dx}{1+x^2+y^2}$.
10. Find the area bounded by one loop of the lemniscates $r^2=a^2$ cos 2ɵ by double integration.
11. If A is the region representating the projection of a surface S on the zx plane, write the formula for the surface area of S.
12. Prove that $_0\int^2 {}_0\int^1 {}_{-1}\int^1$ $(x^2+y^2+z^2)$dz dy dx=6.
13. Find the total work done by a force F=2xyi-4zj+5xk along the curve $x=t^2$, y=2t+1, $z=t^3$ from t=t, t=2.
14. F=e^x sin yi+ex cosyj, evaluate ∫-›F. -›dr along the line joining form (0,0) ti (1,0).
15. State Gauss Divergence theorem.
16. Evaluate using Green's theorem $_c\int x^2$-2xy)dx+(x^2+3)dy around the boundary of the region defined by y^2=8x and x=2.
17. Using Stokes theorem Prove that $_c\int r^{\rightarrow}.dr^{\rightarrow}=0$.
18. Write the Euler's equation when f does not contain y explicity.
19. Show that the Euler's equation for the extremism of $_{x1}\int^{x2}$ $(y^2+y^{12}+2ye^x)$dx reduces to $y^{11}-y=e^x$.
20. Prove that the shortest distance between two points in a plane is along a straight line.

II. Answer any four of the following: 4x5=20

1. Prove that the set V={z+b√2|a,bϵQ}, Q the field of rationals forms a vector space w.r.t addition and multiplication of rational numbers.
2. Prove that:
   i) the subset W={(x1,x2, x3)|x_1^2 + $x^2{}_2$+x3=0} of the vector space V_3 (R) is a subspace of V_3 (R).
   ii) the subset W={(x1,x2,x3)|x12+x22=$x^2{}_3$≤1} of V_3 (R) is not subspace of V_3(R).
3. Find the dimension and basis of the subspace spanned by (1,2,3,4), (1,5-2,4), (1,3,2,4) and (1,6,-3,4) in V4 (R).

4. If matrix of T w.r.t basis B1 and B2 is [-1 2 1 where B1={(1,2,0), (0,-1,0), (1-1,1)},

B2={(1,0), (2,-1)} then find T (x,y,z).

5. Let T:u-›v be alinear transformation. Then prove that (i) R (T) is a subspace of V. (ii) N(T) is a subspace of u.
6. Find all eigen values and a basis for each eigen space of the linear transformation $T:R^3 \to R^3$ defined by T(x,y,z)=(x+y+z, 2y+z, 2y+3z).

III. Answer any three of the following: 3x5=15.

1. Prove that $\int_c (x^2-y^2)\, dx+x^3y\, dy=56\pi$ where C is the semicircle with centre (0,4) and radius 2 units.
2. Evaluate $\int_R xy(x+y)dx\, dy$ over tha region R bounded between parabola $y=x^2$ and the line y=x.
3. Evaluate $\int_o^a \int_y^a$ oy $\frac{xdxdy}{x^2+y^2}$ by changing the order of integration.
4. Find the volume of the tetrahedron bounded by the planes x=0, y=0, z=0 and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$
5. Find the volume common to the sphere $x^2+y^2+z^2=a^2$ and the cylinders $x^2+y^2=ax$.

IV. Answer any three of the following: 3x5=15

1. Evaluate $\int_s \vec{F}.\hat{n}ds$ where S denotes the part of the plane 2x+y+2z=6 which lies in the positive octant and $\vec{F}=4xi+yj+z^2k$.
2. State and prove Green's theorem in the plane.
3. Evaluate ∭div F dv where F $=2x^2yi-y^2j+4xz2k$ and V is region in the first octant bounded by the cylinder $y^2+z^2=9$ and x=2.
4. Using divergence theorem evaluate $\int_s \int_2 \vec{F}$ nds where $F=4xi-2y^2j+z2k$ and S is the surface enclosing the region for which $x^2+y^2\leq 4$ and $0\leq z\leq 3$.
5. Verify Stoke's theorem for the vector field $\vec{F}=(x^2-y^2)i+2xyj$ over the rectangular box bounded by the planes x=0, x=a, y=0, y=b, z=0, z=c with the face z=0 removed.

V. Answer any two of the following: 2x5=10

7. Prove the necessary condition for the integral $I=\int_{x1}^{x2}$-x1f(x, y,y^1)dx where y(x1)=y1 and y(x2)=y2 to be an extremum is that $\frac{\partial f}{\partial y} + \frac{d}{\partial x} = \left(\frac{\partial f}{\partial y^1}\right) = 0$.

1. Find the extresmal of the functional $I=\int_o^4 \sqrt{y}(1+y^{12})\, dx$;y(0)=1, (4)=5.
2. Find the geodesics on a right circular cone.
3. Show that the extremal of the functional $\int_o^2 \sqrt{1+(y^1)^2}dx$ subject to the constraint $\int_o^2$ ydx=π/2 and end conditions y(0)=0, y(2)-o is a circular arc.

BANGALORE UNIVERSITY

VI Semester B.Sc.,

MATHEMATICS (Paper VIII)

Model Paper-I

Time: 3 hours Max. Marks: 90

I. Answer any fifteen questions: 2x15=30

1. Find the locus of the point z, satisfying |z-i/z+i| >2.
2. Evaluate $\lim z \to e^{i\pi/4} \dfrac{z^2}{Z^4+z^2+1}$.
3. Show that $f(z)=e^y(\cos x+i\sin x)$ is not analytic.
4. Find the harmonic conjugate of $u=e^x \sin y+x^2-y^2$.
4. Show that $w=e^2$ is an conformal transformation.
5. Find the fixed points of the transformation $w= \dfrac{1-z}{1+z}$
6. Evaluate $_{(0,1)}\int^{(2,5)}(3x+y)dx+(xy-x)dy$ along the curve $y=x^2+1$.
7. State Cauchy's integral formula.
8. Evaluate $_c\int \dfrac{e^{2z}d}{cz+2i}$ where C is the unit circle with centre at origin
9. State Lovell's theorem.
10. Prove that F{f (t-a)]= e^(iצa) f(צ).
11. Write(i) cosine form of Fourier integral, (ii) sine form of Fourier Integral.
12. Find the Fourier cosine transform of $f(x)=\{x_0$ 0<x<2 otherwise
13. Find the Fourier sine transform of f(x)=1/x, x>0.
14. Prove that F_s[f'(x)]=-צF_c[f(x)].
15. Using Regula Falsi method, find the fifth root of 10 using $x_0=0,x_1=1$ in two steps.
16. Write the general formula for secant method.
17. Find the greatest value $\left(\dfrac{12}{23}\right)$ using power method. Do two steps only.
18. Using Taylor's series method find y at x=1.1 considering terms up to second degree, given that dy/dx=2+y and y(1) =0.
19. Using Eulers method solve dy dx=x+y with the initial value y(0)=1 for x=0.1 in two steps.

II. Answer any four of the following: 4x5=20

a. Sate and prove the necessary condition for the function f(z) to be analytic.
b. If f(z) is analytic function of z, prove that $\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$ 1R| f(z) ^{2}=2| f1(z)^2|
c. If f(z)=u+iv and u-v=ex (cosy-siny), find f(z) in terms of z.
d. Show that the transformation W=zw transforms the circle |z-a| =r on to a lima on or cardioids.
e. Prove that a bilinear transformation transforms circles into circles or straight lines.
f. Find the bilinear transformation which maps z=∞, I,o on to w= o, i, ∞

III. Answer any two of the following: 2x5=10

1. Evaluate $f_c\int x(x+y)dx+x^2ydy$ along:
   i) the Straight line y=3x from (0,0) to (3,9)

   ii) the parabola $y=x^2$ between (0,0) to (3,9).

2. State and prove Cauchy's integral formula.
3. Evaluate $\int_c \frac{z}{(z^2+1)} \frac{dz}{(z2-9)}$ where C:|z|=2.

4. State and prove Cauchy's inequality.

IV Answer any three of the following 3x5=15

1. Using fourier integral formula, show that $f(x)=\frac{2}{\pi}\int_o^\infty \frac{1-\cos\pi}{s}\sin sx\,ds$ Where f (x) = $\{1_0$ of o0≤x ≤ π o for x > π

2. Find the Fourer transform of (x)={1-|x|o for |x|>| and hence deduce that $\int_o^\infty \frac{\sin^2 t}{t2}dt=\pi/2$

3. Find the Fourier sine transform of e^{-ax}/x, (a>0).
4. Find the Fourier cosine transform of $f(x)=x^{n-1}ax\ x$ (n≥2).
5. Assuming f(x)=o as x→∞, Prove that
   $Fc\{f^1(x)\}=-\sqrt{2/\pi}\, f(o)+ৠ Fs\{f(x)\}$ $Fc\{f^{11}(x)\}=-\sqrt{2/\pi}\, f^1(o)+ৠ Fs\{f(x)\}$

V. Answer any three of the following: 3x5=15

1. Find a real root of the equation $f(x)=x^3-5x+1=0$ lies in the interal (0,1) perform 4 iterations of the secant method.
2. Find a real root of the equation xe^x-=2=0 correct to three decimal place
3. Solve the system of equation by Gaus-Seidel method. 10x+y+z=12,x+10y+z=12,x+y10z-12

4. Use power method to find the largest eigen value of the matrix $A=\begin{bmatrix}-4 & -5\\ 1 & 2\end{bmatrix}$

5. By using Runge-Kutta method. Solve $\frac{dy}{dx}=\frac{y-x}{y+x}$ with Y(0)=1. Compute y(0.2) by taking

   h=0.2