Unsteady Hydro magnetic Heat Transfer flow in a vertical channel with effect of dissipation

S.T.Dinesh Kumar¹, P.Raveendra Nath², P.M.V.Prasad³

ABSTRACT: We make an attempt to investigate the Flow and convective Heat transfer through a porous medium in a vertical channel on whose walls a travelling thermal wave is imposed. A non-linear in nature. By taking into the account the effect of viscous dissipation and the aspect ration δ as a perturbation parameter the governing equations are solved by using Regular perturbation scheme. The velocity, temperature, stress and rate of heat transfer on y = ±L are evaluated numerically for different variation of G, R, D¹, γ and x + γt.

KEYWORDS: Convective heat transfer, Porous medium, viscous dissipation,

1. INTRODUCTION

It is well known that in order to harness maximal geothermal energy one should have complete and precise knowledge of quanta of perturbation needed to initiate convection currents in mineral fluids embedded in the earth’s crest which enables one to use mineral energy to extract the minerals. For example, in the recovery of hydrocarbons from underground petroleum reservoirs, the use of thermal processes is gaining importance to enhance the recovery. Heat can be injected into the reservoir as hot water or steam or heat can be generated by burning part of the reservoir crude. In all such thermal recovery processes fluid flow takes place through a porous medium and convection flow through a porous medium is of utmost importance, determination of the external energy required to initiate convection currents needs a thorough understanding of convective processes in a porous medium. There has been a great quest in Geophysicists to study the problem of convection currents in a porous medium heated from below. More recently, studies of the viscous dissipation effect in laminar duct flows have been performed in order to include the area of slug velocity profile, of slip flow in micro tubes and of Non-Newtonian fluid behavior ([1]-[4]). In view of this several authors notably, Barletta ([5]-[7]), El-hakeing [8], Bulent Yesilata [9], Israel et al [10] Raveendra nath et al[11], have studied the effect of viscous dissipation on the convective flows past on infinite vertical plates and through vertical channels and Ducts. The effect of viscous dissipation on natural convection has been studied for a few different cases including the natural convection from horizontal cylinder.

¹Assistant professor, Department of Mathematics,Govt.Science college,Chitradurgam, Karnataka
²Lecturer in Mathematics, Department of Mathematics,Sri Krishnadevaraya University College of Engineering and Technology,S.K. University, Anantapur - 515 055, A.P., India. E-mail: ravindrasku@gmail.com
³Department of Mathematics, S.V.G.S Degree College, Nellore, A.P., India. E-mail:pmvprasad65@yahoo.com

2. THE PROBLEM FORMULATION

We consider the motion of viscous, fluid through a porous medium in a vertical channel bounded by flat walls. The thermal buoyancy in the flow field is created by a traveling thermal wave imposed on the boundary wall at y=L while the boundary at y = -L is maintained at constant temperature T₁. The walls are maintained at constant concentrations. he Boussinesq approximation is used so that the density variation will be considered only in the buoyancy force. Also the kinematic viscosity v, the thermal conducting k are treated as constants. We choose a rectangular Cartesian system 0 (x,y) with x-axis in the vertical direction and y-axis normal to the walls. The walls of the channel are at y=±L.

The equations governing the unsteady flow and heat transfer are

Equation of linear momentum

\[ \rho_e \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \rho g - \left( \frac{\mu}{k} \right) u \]  

Equation of continuity

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]  

Equation of energy

\[ \rho_e C_p \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \lambda \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \]

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Equation of state
\[ \rho - \rho_e = -\beta \rho_e (T - T_e)^2 \]  
(5)

where \( \rho_e \) is the density of the fluid in the equilibrium state, \( T_e \) is the temperature and in the equilibrium state, \((u,v)\) are the velocity components along \((x,y)\) directions, \( p \) is the pressure, \( T \) is the temperature in the flow region, \( \rho \) is the density of the fluid, \( \mu \) is the constant coefficient of viscosity, \( C_p \) is the specific heat at constant pressure, \( \lambda \) is the coefficient of thermal conductivity, \( k \) is the permeability of the porous medium, \( \beta \) is the coefficient of thermal expansion and \( Q \) is the strength of the constant internal heat source.

In the equilibrium state
\[ 0 = \left( \frac{\partial p_e}{\partial x} \right) - \rho_e g \]  
(6)

where \( p = p_e + p_D \) being the hydrodynamic pressure. The flow is maintained by a constant volume flux for which a characteristic velocity is defined as
\[ Q = \frac{1}{2L} \int u \, d\gamma \]  
(7)

The boundary conditions for the velocity and temperature fields are
\[ u = 0, \, v = 0, \, T = T_1 \text{ on } y = -L \]  
(8)

where \( \Delta T_e = T_2 - T_1 \) and \( \sin(mx + nt) \) is the imposed traveling thermal wave.

In view of the continuity equation we define the stream function \( \psi \) as
\[ u = -\psi_y, \, v = \psi_x \]  
(10)

Eliminating pressure \( p \) from equations (2)&(3) and using the equations governing the flow in terms of \( \psi \) are2
\[ \left[ (\nabla^2 \psi), + \psi_x (\nabla^2 \psi)_y - \psi_y (\nabla^2 \psi)_x \right] \
\[ = \nu \nabla^4 \psi - \beta g(T - T_0), - \left( \frac{v}{k} \right) \nabla^2 \psi \]  
(11)

\[ \rho_e C_p \left( \frac{\partial \theta}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} \right) = \lambda \nabla^2 \theta \]  
(12)

Introducing the non-dimensional variables in (2.10)&(2.11)
\[ x' = mx, \, y' = \frac{y}{L}, \, t' = tv/L^2, \]  
\[ \Psi' = \frac{\Psi}{v} \]  
\[ \theta = \frac{T - T_e}{\Delta T_e} \]  
(13)

(Under the equilibrium state)
\[ \Delta T_e = T_e(L) - T_e(-L) = \frac{QL^2}{\lambda} \]

The governing equations in the non-dimensional form (after dropping the dashes) are
\[ \delta R \left( \delta (\nabla_1^2 \psi) + \frac{\partial \psi}{\partial \theta} \frac{\partial \theta}{\partial (x,y)} \right) \]
\[ = \nabla_1^4 \psi + \frac{2G}{R} (\theta \theta_y) - \Delta^2 \nabla_1^2 \theta \]  
(14)

The energy equation in the non-dimensional form is
\[ \delta P \left( \delta \frac{\partial \psi}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} \right) = \nabla_1^2 \theta \]  
(15)

Where
\[ R = \frac{UL}{v} \]  
(Reynolds number)
\[ G = \frac{\beta g \Delta T_e L^3}{\nu^2} \]  
(Grashof number)
\[ p = \frac{\mu c_p}{k} \]  
(Prandtl number),
\[ D = \frac{L^2}{k} \]  
(Darcy parameter),
\[ \delta = mL \]  
(Asspect ratio)
\[ \gamma = \frac{n}{vm^2} \]  
(Non-dimensional thermal wave velocity)
\[ \nabla_1^2 \psi = \delta^2 \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \]

The corresponding boundary conditions are
\[ \psi(+1) = \psi(-1) = 1 \]  
(16)

\[ \frac{\partial \psi}{\partial x} = 0, \, \frac{\partial \psi}{\partial y} = 0 \text{ at } y = \pm 1 \]  
(17)

\[ \theta(x, y) = 1 \text{ on } y = -1 \]  
(18)

\[ \theta(x, y) = \sin(x + \gamma) \text{ on } y = 1 \]  
(19)

\[ \frac{\partial \theta}{\partial y} = 0 \text{ at } y = 0 \]  
(20)

The value of \( \psi \) on the boundary assumes the constant volumetric flow in consistent with the hypothesis. Also the wall temperature varies in the axial direction in accordance with the prescribed arbitrary function \( t \).
3. SHEAR STRESS AND NUSSELT NUMBER

The shear stress on the channel walls is given by
\[
\tau = \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)_{y=\pm L}
\]
Which in the non-dimensional form reduces to
\[
\tau = \left( \frac{\mu U}{a} \right) = \left( y_{yy} - \delta^2 y_{xx} \right)
\]
\[
= \left[ y_{0,yy} + \delta y_{1,yy} + O(\delta^2) \right]_{y = \pm 1}
\]
And the corresponding expressions are
\[
(\tau)_{y = +1} = a_{99} + \delta (a_{81} + a_{82} Ch(M_1))
\]
\[
Cth(M_1) + a_{83} Ch(M_1) + a_{84} Sh(M_1) + a_{85}
\]
(\tau)_{y = -1} = a_{90} + \delta (a_{90} + a_{91} Ch(M_1))
\[
Cth(M_1) + a_{88} Ch(M_1) + a_{89} Sh(M_1) + a_{90}
\]
The local rate of heat transfer coefficient (Nusselt number \(Nu\)) on the walls has been calculated using the formula
\[
Nu = \frac{1}{\theta_m - \theta_w} \left( \frac{\partial \theta}{\partial y} \right)_{y = \pm 1}
\]
and the corresponding expressions are
\[
(Nu)_{y = +1} = \frac{(a_{93} + \delta a_{95})}{(a_{97} - \sin(D_y))}
\]
\[
(Nu)_{y = -1} = \frac{(a_{94} + \delta a_{96})}{(a_{97} - 1)}
\]
where \(a_{95}, \ldots, a_{97}\) are constants

4. DISCUSSION OF THE NUMERICAL RESULTS

In this analysis we discuss the Convective Heat transfer through a porous medium confined in a vertical channel in whose walls traveling a thermal wavy is imposed with quadratic density temperature variation and viscous dissipation. The Non-dimensional temperature \(\theta\) is exhibited in Figs.(1-4) for different values of \(G, D^{-1}, R, \gamma\) and \(x+\gamma t\). It is found from Fig.1 shows that the actual temperature depreciates in the right half with increase in \(G \leq 2 \times 10^3\) and for higher \(G \geq 3 \times 10^3\),the actual temperature experiences a reduction in the entire flow region. An increase in \(G<0\) enhance the actual temperature everywhere in the flow region.

Fig.1 Variation of \(\theta\) with \(G\)
\[D^{-1}=10^3, x+\gamma t=\pi/4, R=35\]
The behaviour of \(\theta\) with \(D^{-1}\) shows that lesser the permeability of the porous medium larger the actual temperature and for further lowering of the permeability smaller the actual temperature in the flow field. Also an increase in \(\gamma\) enhances \(\theta\) in the flow region (Fig.2).

Fig.2 Variation of \(\theta\) with \(D^{-1}\)
\[G=10^3, x+\gamma t=\pi/4, R=35\]
The behaviour of \(\theta\) with phase \(x+\gamma t\) of the boundary temperature enhances with increase in \(x+\gamma t < \pi/2\) and depreciates for further increase in \(x+\gamma t \geq \pi\) except in a narrow region adjacent to \(y = 1\) in which the temperature experiences an enhancement (Fig.4).

Fig.4 Variation of \(\theta\) with \(x+\gamma t\)
\[D^{-1}=10^3, R=35, G=10^3\]
From Fig.3 we find that higher the Reynolds number \(R\) smaller the actual temperature in the flow region.
Fig. 3  Variation of \( \theta \) with R

\[ D^1 = 10^3 \times x + \gamma t = \pi/4, R = 35, G = 10^3 \]

The Nusselt Number (Nu) which measures the rate of heat transfer at the walls is shown in Tables (1-4) for different values of G, D^1, R, \( \gamma \), x + \( \gamma \)t. It is found that the rate of heat transfer depreciates with increase in G>0 and enhances with G<0. Lesser the permeability of the porous medium smaller the magnitude of Nu at y = -1 and larger at y = 1. An increase in the thermal wave velocity \( \gamma \) reduces \( |Nu| \) at both the walls. The variation of Nu with Reynolds number R shows that at y = -1 the rate of heat transfer reduces with R for \( |G| \leq 10^3 \) and enhances for higher \( |G| \geq 10^3 \), while at y = 1 Nu experiences a depreciation with R for all G (Tables 1 and 2). The rate of heat transfer reduces with increase in the phase \( x + \gamma t \geq \pi/2 \) and enhances with higher values of \( x + \gamma t \geq \pi \) in both heating and cooling cases (Tables 3 and 4).

### REFERENCES


