A Comparative Study of Magnetite and Mn–Zn Ferrite Nanoliquids Flow Inspired by Nonlinear Thermal Radiation

P. B. Sampath Kumar1, B. Mahanthesh1,2,*, and B. Phalaksha Murthy1,3

1Department of Studies and Research in Mathematics, Kuvempu University, Shankaraghatta, Shimoga, 577451, Karnataka, India
2Department of Mathematics, Christ University, Bangalore 560029, Karnataka, India
3Department of Mathematics, Govt. Science College, Chitradurga 577501, Karnataka, India

The characteristics of the magnetohydrodynamic (MHD) stagnation point flow of ferrofluids are investigated. The effects of nonlinear thermal radiation, heat generation and viscous dissipation are considered. Two different nanoparticles (Fe3O4 and Mn–ZnFe2O4) are comprised in the base fluid (water). The ordinary differential equations are formed using suitable similarity transformations from the governing partial differential equations. The subsequent nonlinear ordinary differential equations are solved numerically using RKF-45 method. The influence of governing parameters on the results are analysed. It is found that the thermal boundary layer thickens due to the influence of nonlinear radiation and heat generation for both the fluids. The rate of heat transfer is higher for Mn–Zn ferrite-nanofluid in comparison with magnetite nanofluid.


1. INTRODUCTION

The thermal properties of the ordinary fluids are not sufficient to meet today’s cooling rate requirements in industrial applications. Therefore, the nanofluids are introduced to enhance the thermal performance of ordinary fluids. Basically, the nanofluids are engineered colloidal suspensions of nanoparticles in a base fluid. The nanoparticles are typically made up of metals (Al, Cu), oxides (Al2O3, TiO2 and CuO), carbides (SiC), nitrides (AlN, SiN) or nonmetals (Graphite, carbon nanotubes) and the base fluids are usually conductive fluids, such as water, ethylene glycol and etc. The term of nanofluid was first introduced by Choi.1 Buongiorno2 proposed seven slip mechanisms to write down conservation equations based on the Brownian diffusion and thermophoresis effects. Philip et al.3 have investigated the tunable thermal property of a magnetically polarizable nanofluid that consists of a colloidal suspension of magnetite nanoparticles. They have confirmed that the large enhancement in thermal conductivity is due to the efficient heat transfer. Shima et al.4,5 have studied on thermal conductivity of nanofluid and they concluded that the thermal conductivity and viscosity are improved in a stable magnetic nanofluid containing particles size < 10 nm. Recently, Gorla and Chamkha6 have analyzed the flow of nanofluid with natural convective boundary layer over a horizontal plate along with porous medium. Rashidi et al.7 have investigated entropy generation in steady MHD flow of nanofluid due to a rotating porous disk. Gireesha et al.8 have analyzed the effect of suspended particles on nanofluid flow and heat transfer over a stretching sheet saturated by a porous medium. The heat and mass transfer of water based nanofluid flow over a stationary/moving vertical plate were explored by Mahanthesh et al.9 In very recently Farooq et al.,10 Hayat et al.,11 Mahanthesh et al.,12 Hayat et al.,13 Kumar et al.14 and Mahanthesh et al.15 were investigated nanofluid flow considered with various aspects.

Besides, the ferrofluid comprises of iron-based nanoparticles such as magnetite, hematite, cobalt ferrite, etc., Researchers and scientists have focused considerably on the surface driven ferrofluid flows owing to their numerous industrial and biomedical demands. For instance, iron-based nanoparticles can be used for efficient drug delivery by guiding the particles via external magnets; magnetic nanoparticles are prominent in hyperthermia. Several researchers investigated diversified characteristics of such ferrofluid problems. For instance, Tangthieng et al.16
addressed heat transfer enhancement in ferrofluids subjected to steady magnetic fields. Jue used semi-implicit finite element method in order to simulate magnetic gradient and thermal buoyancy induced cavity ferrofluid flow. Nanjundappa et al. analyzed the influence of magnetic field dependent viscosity on the horizontal layer of ferrofluid. Muthukumaran et al. reported the experimental results for highly stable magnetite nanoparticle suspensions of different size and magnetization by controlling the reaction temperature and enhanced thermal stability of nanoparticles. Sheikhholeslami and Ganji have investigated the MHD flow and heat transfer of ferrofluid with the effect of convective heat transfer. The stagnation point flow and heat transfer of ferrofluid towards a stretching sheet in the presence of viscous dissipation were investigated by Zafar et al. They have considered three types of ferroparticles magnetite (Fe3O4), cobalt ferrite (CoFe2O4) and Mn–Zn ferrite (Mn–ZnFe2O4) with water and kerosene as conventional base fluids. Recently Sheikhholeslami, Rashidi and Ganji have studied the effect of magnetic field with suspended ferroparticles.

The fluid flow at a stagnation point over a stretching sheet is crucial in theoretical and application point of view in fluid dynamics. Chaim was the first to investigate the stagnation-point flow towards a stretching sheet. Mahapatra and Gupta have studied the similar problem by considering the strain-rate and the stretching rate to be different. They obtained boundary layer adjacent to the sheet which completely depends on the ratio between the strain-rate of stagnation-point flow and the stretching rate of the sheet. The steady and unsteady stagnation-point flow of an incompressible viscous fluid over a stretching surface was studied by Paulet and Weidman. The stagnation point flow of an electrically conducting fluid over a stretching surface under the influence of magnetic field was investigated by Mahapatra et al. Pal et al. have analyzed the nanofluid flow and non-isothermal heat transfer at the stagnation-point over a stretching/shrinking sheet embedded in a porous medium. Gireesha et al. have presented the numerical solution for boundary layer stagnation-point flow past a stretched surface with melting effect and aligned magnetic field. They have incorporated the nanofluid model by considering Brownian motion with thermophoresis mechanisms. Furthermore, the stagnation-point flow over stretching sheet under different physical aspects were discussed by Ishak, Mabood, Makinde and Hayat et al.

In view of the above discussion, the objectives of present analysis are three folds. Firstly to model the two-dimensional stagnation flow of Magneto-ferrofluids induced by a stretched surface. Secondly to analyze the heat transfer process under the influence of non-linear thermal radiation, heat generation and viscous dissipation effects. Thirdly to compare the flow characteristics of two ferrofluids namely Fe3O4 and Mn–ZnFe2O4 water based nanofluids. Numerical solutions are computed for the governing nonlinear boundary value problem via similarity method.

2. PROBLEM FORMULATION

Consider a steady, two-dimensional boundary layer flow of an incompressible ferrofluid driven by stretching of sheet at \( y = 0 \) with a fixed stagnation point. The \( x \)-axis taken along the sheet and \( y \)-axis is normal to it. The fluid occupies the half-plane \( y > 0 \). Two equal and opposite forces are applied along the sheet, so that the sheet is stretched, keeping the position of the origin unchanged (see Fig. 1). The fluid is assumed to be electrically conducting. The magnetic field of strength \( B_0 \) is applied along \( y \)-direction. The thermophysical properties of the base fluids (water) and the ferroparticles (magnetite and Mn–Zn ferrite) are presented in Table I. The sheet is maintained at constant temperature \( T_w \) whereas \( T_\infty \) denotes the temperature outside the thermal boundary layer. The velocity of the stretching rate and flow external to the boundary layer are \( U_s(x) = cx \) and \( U_e(x) = ax \), where \( a \) and \( c \) are positive constants.

In terms of stream function \( \psi(x, y) \) such that \( u = \partial \psi / \partial y \) and \( v = - (\partial \psi / \partial x) \), the governing equations are

\[
\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial x \partial y} = 0 \tag{1}
\]

\[
\frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x \partial y} = \frac{1}{\rho} \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\nu} \frac{\partial^2 \psi}{\partial y^2} \tag{2}
\]

\[
\frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} = \alpha \left( \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho \nu} \frac{\partial^2 \psi}{\partial y^2} \right)^2 - \frac{1}{\rho (\kappa_c \nu)} \frac{\partial^2 \psi}{\partial y^2} Q_0 \left( T - T_\infty \right) \tag{3}
\]

where \( u \) and \( v \) are the velocity components along the \( x \)- and \( y \)-axes, \( \mu_d \)-dynamic viscosity of nanofluid, \( \sigma \)-the electrical conductivity, \( B_0 \)-magnetic field, \( \nu_d \)-effective kinematic viscosity of nanofluid, and \( \alpha_d \)-effective

![Flow configuration and coordinate system.](image)

Fig. 1. Flow configuration and coordinate system.
We look for a similarity solution of (1)–(6) of the following form:
\[ \eta = \frac{\sqrt{c}}{\sqrt[3]{v'}} y, \quad f(\eta) = \frac{\psi}{\sqrt[3]{v' U_w}}, \]
where \( \theta_u = T_u/T_\infty \), \( \theta_u > 1 \) is the temperature ratio parameter. Following nonlinear system of ordinary differential equations are obtained from the Eqs. (2) and (6) by employing similarity variables (8);
\[ f''''(\eta) + (1 - \phi^2)^{3/2} \left\{ \left( 1 - \phi + \phi \frac{\rho_s}{\rho_f} \right) f''(\eta) \right\} = 0 \]
\[ -f'(\eta)^2 + \delta^2 M(\delta - f'(\eta)) = 0 \]
The radiative heat flux expression in Eq. (3) is given by the Rosseland approximation as follows;\(^{11,33}\)
\[ q_1 = \frac{4\sigma^* T^4}{3 k^*} = \frac{3}{16 \sigma^*} \frac{\partial T}{\partial y} \]
where \( \sigma^* \) and \( k^* \) are the Stefan-Boltzmann constant and the mean absorption coefficient respectively. In view of Eq. (5), the Eq. (3) reduces to
\[ \frac{\partial^2 T}{\partial y^2} \frac{\partial^2 T}{\partial x^2} = \frac{16 \sigma^*}{3 (\rho c_p)_a} \left[ T^4 \frac{\partial^2 T}{\partial y^2} + 3 T^2 \left( \frac{\partial T}{\partial y} \right)^2 \right] + \frac{Q_0}{(\rho c_p)_a} (T - T_w) \]
The effective properties of nanofluids may be expressed in terms of the properties of base fluid and nanoparticle and the solid volume fraction of nanofluid as follows;
\[ n_u = \frac{\mu_u}{\rho u}, \quad \mu_f = \frac{\mu_f}{(1 - \phi)^{2/3}}, \]
\[ \rho_u = (1 - \phi) \rho_f + \phi \rho_s, \]
\[ (\rho c_p)_a = (1 - \phi) (\rho c_p)_f + \phi (\rho c_p)_s, \]
\[ k_u = \frac{(k_u + 2 k_f)}{(k_u + 2 k_f)} - 2 \phi \left( k_f - k_u \right) \]
here \( k_u \)—the thermal conductivity of the nanoparticle, \( k_f \)—thermal conductivity of the base fluid, \( k_u \)—thermal conductivity of solid nanoparticles, \( \sigma_f \) the electrical conductivity of the base fluid, \( \sigma_s \) the electrical conductivity of the nanoparticle, the subscripts \( s \) and \( f \) denotes to the solid and fluid respectively and \( \phi \)—solid volume fraction of nanofluid.

### Table I. Thermophysical properties of base fluid and magnetic nanoparticles.

<table>
<thead>
<tr>
<th></th>
<th>( k ) (W/m-K)</th>
<th>( \rho ) (kg/m³)</th>
<th>( c_p ) (J/kgK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>0.613</td>
<td>997.1</td>
<td>4179</td>
</tr>
<tr>
<td>FeO₂</td>
<td>9.7</td>
<td>5180</td>
<td>670</td>
</tr>
<tr>
<td>Mn-ZnFe₂O₄</td>
<td>5</td>
<td>4900</td>
<td>800</td>
</tr>
</tbody>
</table>

thermal diffusivity of nanofluid, \( T \)—fluid temperature, \( T_a \)—ambient fluid temperature, \( n_u - \) heat capacity of nanofluid, \( Q_{nf} \)—dimensional heat generation/absorption coefficient and \( c_p \)—the specific heat.

The boundary conditions for the problem are given by;
\[ \frac{\partial \psi}{\partial y} = U_w(x), \quad \frac{\partial \psi}{\partial x} = 0, \quad T = T_w, \quad \text{at} \quad y = 0 \]
\[ \frac{\partial \psi}{\partial y} \rightarrow U_w(x), \quad T \rightarrow T_\infty, \quad \text{as} \quad y \rightarrow \infty \]

Subjected to the boundary condition;
\[ q(0) = 0, \quad f(0) = 1, \quad \theta(0) = 1 \quad \text{at} \quad y = 0 \]
\[ f'(\eta) \rightarrow \delta, \quad \theta(\eta) \rightarrow 0 \quad \text{as} \quad \eta \rightarrow \infty \]
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Table II. Variation of \(-\sqrt{\mu_0\mu_r C_{fl}}\) with solid volume fraction for different values of magnetic and stretching parameters with \(Ec = 0.2\), \(Q = 0.5\), \(R = 1\), \(\theta_e = 1.5\).

<table>
<thead>
<tr>
<th>(M)</th>
<th>(\phi)</th>
<th>(\delta = 0.1)</th>
<th>(\delta = 0.3)</th>
<th>(\delta = 0.5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.1</td>
<td>1.00207</td>
<td>0.87801</td>
<td>0.68973</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>1.31759</td>
<td>1.15450</td>
<td>0.90692</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>1.73762</td>
<td>1.52252</td>
<td>1.19602</td>
</tr>
<tr>
<td>0.05</td>
<td>0.1</td>
<td>1.00072</td>
<td>0.87683</td>
<td>0.68880</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>1.30450</td>
<td>1.14302</td>
<td>0.89790</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>1.71089</td>
<td>1.49909</td>
<td>1.17761</td>
</tr>
</tbody>
</table>

3. RESULTS AND DISCUSSION

The coupled ordinary differential nonlinear Eqs. (9) and (10) along with the boundary conditions (11) are solved numerically using Runge-Kutta-Fehlberg 4–5th order scheme coupled with shooting technique. The integration length varies with respect to the physical parameter values and it has been suitably chosen each time such that the boundary conditions at the outer edge of the boundary layer are satisfied. Throughout our computation the volume fraction of nanoparticle is considered in the range \(0 \leq \phi \leq 0.2\) while the Prandtl number \(Pr = 6.2\) for water. For water-based ferrofluids, the variations of skin friction with solid volume fraction for different values of stretching and magnetic parameter are shown in Table II. It is observed that the skin friction is smaller in the absence of magnetic field. Also, the skin friction coefficient increases with increase in volume fraction parameter. Whereas the skin friction coefficient decreases by increasing stretching parameter. Table III reveals that the Nusselt number is higher for nonlinear radiation influence than the linear radiation. Further, the Nusselt number increases with volume fraction parameter and it is decreased by increasing stretching parameter. It is interesting to note that \(Fe_3O_4\) nanoparticle has higher skin friction coefficient than \(Mn-ZnFe_2O_4\) nanoparticle.

Figures 2–14 are plotted to bring out the salient features of different flow fields versus different values of physical parameters. In these figures, red lines indicate the profiles for \(Fe_3O_4\)-nanoparticle and green lines indicate the profiles for \(Mn-ZnFe_2O_4\) nanoparticle. The velocity profile for different values of stretching parameter \(\delta\) is presented in Figure 2. It is observed that the flow has a boundary layer structure when the stretching velocity is less than the free stream velocity (\(\delta = a/c > 1\)). Physically, the stretching motion near the stagnation point increased to accelerate velocity of the external stream, which leads to an increase in the thickness of the boundary layer with \(\delta\). However the stretching velocity \(cx\) of the surface exceeds the free stream velocity \(ax\) (\(\delta < 1\)), therefore inverted boundary layer structure is formed and for \(\delta = 1\) there is no boundary layer formation because the stretching velocity is equal to the free stream velocity. The velocity profile for different values of magnetic parameter is displayed in Figure 3. Because of the influence of Lorentz force the velocity field and its associated boundary layer thickness decreases. The variation of stretching ratio parameter \(\delta\) on velocity and temperature profiles respectively plotted in Figures 4 and 5. These figures indicate that, the velocity field increases and temperature field decreases with an increase in stretching ratio parameter.

Figure 6 exhibit the effect of volume fraction parameter on temperature profiles. It is observed that the dimensionless temperature is found to be higher and its corresponding thermal boundary layer become thicker. Figures 7 and 8 demonstrate the effects of heat source and nonlinear.

Table III. Variation of \(Nu/(\sqrt{\kappa_\sigma})\) with solid volume fraction for different values of magnetic and stretching parameters with \(Ec = 0.2\), \(Q = 0.5\), \(M = 0.5\), \(R = 1\).

<table>
<thead>
<tr>
<th>(\phi)</th>
<th>(\delta = 0.1)</th>
<th>(\delta = 0.3)</th>
<th>(\delta = 0.5)</th>
<th>(\delta = 0.1)</th>
<th>(\delta = 0.3)</th>
<th>(\delta = 0.5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Fe_3O_4)</td>
<td>0.01</td>
<td>3.951494</td>
<td>-3.048904</td>
<td>-7.476607</td>
<td>5.509048</td>
<td>-1.401750</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>7.128524</td>
<td>-1.227471</td>
<td>-6.073685</td>
<td>8.908286</td>
<td>-0.293510</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>9.857402</td>
<td>0.395685</td>
<td>-4.752053</td>
<td>12.07021</td>
<td>0.724722</td>
</tr>
<tr>
<td>(Mn-ZnFe_2O_4)</td>
<td>0.01</td>
<td>2.097975</td>
<td>-1.681712</td>
<td>-4.091306</td>
<td>3.240191</td>
<td>-0.863060</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>3.305423</td>
<td>-0.926465</td>
<td>-3.509962</td>
<td>4.423531</td>
<td>-0.384430</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>4.179113</td>
<td>-0.259601</td>
<td>-2.962651</td>
<td>5.057244</td>
<td>0.028913</td>
</tr>
</tbody>
</table>
radiation parameter on temperature profiles. It is observed that the temperature is augmented throughout the boundary layer region as $Q$ and $R$ increases. This is because the heat generation and radiation parameter provides more heat into the fluid, which leads to an intensification of the thermal boundary layer. Additionally, it is noted that the non-linear thermal radiation effect is more prominent on temperature field than that of linear thermal radiation. Also one can observe that the temperature profile is smaller for Mn–ZnFe$_2$O$_4$-nanofluid in comparison with Fe$_3$O$_4$-nanofluid. The variation of temperature profile $\theta(\eta)$ for different values of temperature ratio parameter is plotted in Figure 9. It is found that the thermal boundary layer thickness increases by increasing the temperature ratio parameter. This is happen due to the enhanced thermal conductivity of the flow.

The variation of skin friction coefficient for different values of the magnetic parameter and Eckert number is explained in Figure 10. As expected, the skin friction decreases for strong magnetic field. The variation of skin friction for different values of magnetic parameter and stretching parameter is presented in Figure 11.
for Mn–ZnFe₂O₃-nanofluid in comparison with Fe₂O₃-nanofluid. The Nusselt number profile for various values of the magnetic parameter, Eckert number and stretching parameter is presented in Figures 12 and 13. It is seen that the Nusselt number is reduced when the magnetic parameter increases. Further, we observed that the response of the Nusselt number is opposite for larger values of the Eckert number and stretching parameter. Finally, the effect

**Fig. 8.** Effect of nonlinear radiation parameter on temperature profile.

**Fig. 9.** Effect of temperature ratio parameter on temperature profile.

**Fig. 10.** Variation of magnetic parameter with Eckert number on skin friction coefficient.

**Fig. 11.** Variation of magnetic parameter with stretching parameter on skin friction coefficient.

**Fig. 12.** Variation of magnetic parameter with Eckert number on Nusselt number.

**Fig. 13.** Variation of magnetic parameter with stretching parameter on Nusselt number.
• The dimensionless velocity decreases with increase in stretching parameter.
• The dimensionless temperature decreases with increasing stretching parameter.
• The nonlinear thermal radiation plays a crucial role in cooling and heating process.
• The skin friction coefficient and Nusselt number decreased for the strong magnetic field.
• The Nusselt number increases with nonlinear thermal radiation parameter.
• Finally, comparison of Magnetite and Mn–Zn ferrite-water nanofluids, it can be concluded that magnetite have a higher rate of skin friction and Nusselt numbers.

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