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SIIIS - N 195 B - 16

B.A./B.Sc. IIIrd Semester Degree Examination

Mathematics

(Groups, Rings, Line and Multiple Integrals)

Paper : 3.1

(New)

Time : 3 Hours Maximum Marks : 60

Instructions to Candidates :

Answer **all** questions from all Sections.

Section - A

I. Answer any **"TEN"** of the following **(10×2=20)**

1) Write the associative law and commutative law of a group G under binary operation "$\cdot$"
2) Show that the multiplicative group of fourth root of unity $\{1,-1,i,-i\}$ is a cyclic group.
3) Define the order of an element of a group G.
4) State "FERMATS" Theorem
5) Define the Isomorphism of two groups G & G'.
6) Define the Kernel of homomorphism
7) Write the left & write distributive laws of ring R.
8) Define the field
9) Evaluate $\int_c [x-y]\,dx+[x+y]\,dy$ along the Curve $c: x=t^2; y=t$ where ; $0 \le t \le 1$.
10) Evaluate $\int_c (2y+x)\,dx+(3x-y)\,dy$ along the Curve $x=2t;\ y=t^2$ where ; $0 \le t \le 1$.
11) Evaluate $\int_0^a\int_0^b \left[x^2+y^2\right] dx\,dy$
12) Evaluate $\int_0^1\int_1^2 \left[x^3+y^3\right] dy\,dx$

Section - B

II. Answer any **Five** of the following **(5×5=25)**

1) State and Prove the necessary and sufficient Condition for a sub Group "H" of a group 'G'
2) If $a,b \in G$ then Prove the following results.
   i) If $a \in G$, then prove that $\left(a^{-1}\right)^{-1}=a$
   ii) If $a,b \in G$, then Prove that $(ab)^{-1}=b^{-1}a^{-1}$
3) State and Prove "LAGRANGE'S" Theorem.

4) Prove that the set G/H of all cosets of a Normal. Sub group H of the Group G is a group under the Binary operation defined by. $Ha.Hb = Hab; \forall Ha, Hb \in G/H$

5) Let $f: G \to G'$ be a Homomorphism of a groups with Kernel 'K'. Then Prove the following.

i) K is sub-group of G.

ii) K is normal Sub-grouping

6) Let (R "+") = Group of all real numbers With Respect to addition (R^+ ".") = Group of all positive real numbers with respect to Multiplication then Show that the Mapping $f: R_{,+} \to R_.^+$ defined by $f(x) = e^x$

i) f is homomorphism.

ii) f is 1-1

iii) f is onto.

7) Show that the set $Z_6 = \{0,1,2,3,4,5\}$ has

i) Additive Inverse find?

ii) Satisfies commutative law with respect to addition & multiplication modulo 6+,x

8) Prove that Intersection of two subrings is again a subring of R

Section - C

III. Answer any **THREE** of the following **(3×5=15)**

1) Evaluate $\int_c \left[x^2 - 2y\right] dx + \left[x^2 y + 3\right] dy$. Around the boundary of the region defined by $y^2 = 4x$ and $x = 2$.

2) Prove that $\int_c xydx + yzdy + zxdz$ Where C is the curve defined by $x = t;\ y = t^2; Z = t^3$ and $-1 \le t \le 1$ is $-10/7$.

3) Prove that $\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} \frac{dxdydz}{\sqrt{a^2 - x^2 - y^2 - z^2}} = \frac{\pi a^2}{8}$

4) Find the area of a circle by double integration method whose equation is given by $x^2 + y^2 = 9$

5) By applying the Leibnitz's rule for the differentiation under the integral sign prove that $\int_0^1 \frac{x^{\infty}-1}{\log x} dx = \log(1+\infty)$ where "∞" is any parameter.