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SIVS - N 194 A-17
B.A./B.Sc. IVth Semester Degree Examination
Mathematics
(Vector Analysis and Fourier Transformations)
Paper : 4.2
(New)

Time : 3 Hours

Maximum Marks : 60

Instructions to Candidates :

Answer all sections.

SECTION - A

I. Answer any TEN of the following :

(10 × 2 = 20)

- 1) Define vector differential operator.
- 2) Show that :
 - i) $\nabla \cdot \vec{r} = 3$
 - ii) $\nabla \times \vec{r} = 0$ where \vec{r} is position vector.
- 3) If $\vec{f} = 2x^2z\vec{i} - 10xyz\vec{j} + 3xz^2\vec{k}$, prove that \vec{f} is solenoidal.
- 4) If $\vec{f} = (y+z)\vec{i} + (x-z)\vec{j} + (x-y)\vec{k}$, prove that \vec{f} is irrotational.
- 5) Evaluate $\int_s \vec{r} \times \vec{n} ds$, for any closed surface s .
- 6) Evaluate $\int_c e^x dx + 2ydy - dz$, by stokes theorem where c is curve $x^2 + y^2 = 4, z = 2$.
- 7) If $f(x) = x$ in $[0, \pi]$, find Fourier coefficient a_n .
- 8) If $f(x) = e^{-x}$ in $[-\pi, \pi]$, find Fourier constant a_0 .

- 9) Define Harmonic analysis.
- 10) Define Infinite Fourier series.
- 11) Find the sine transform of $f(x) = e^{-2x}$.

12) Find the inverse cosine of $f(u) = \begin{cases} 1-u/2 & ; 0 \leq u \leq 2 \\ 0 & ; u > 2 \end{cases}$.

SECTION - B

II Answer any **Four** of the following.

(4 × 5 = 20)

- 1) Find the directional derivative of $\phi(x, y, z) = \frac{1}{x^2 + y^2 + z^2}$ at $(1, 2, -3)$ in the direction of $2i - 3j + k$.
- 2) If $\vec{r} = xi + yj + zk$, such that $r = |\vec{r}|$ then show that $\text{curl} (f(r)\vec{r}) = 0$.
- 3) Prove that $\text{curl} (F \times G) = (\text{div} G) F + (G \cdot \nabla) F - (\text{div} F) G - (F \cdot \nabla) G$ where F & G are continuously differentiable vector point functions.
- 4) If $\vec{f} = x^2yi + y^2zj + z^2xk$. Find $\text{curl} (\text{curl} \vec{f})$ at $(1, 0, 1)$.
- 5) Verify Greens theorem for $\int_c [(3x^2 - 8y^2)dx + (4y - 6xy)dy]$
where c is the boundary of the region between the curves $x = y, y = \sqrt{x}$.
- 6) Verify divergence theorem for $\vec{f} = y^2z^2i + z^2x^2j + x^2y^2k$, over the surface S of a hemisphere $x \geq 0, x^2 + y^2 + z^2 = 1$.

SECTION - C

III Answer any **Four** of the following questions.

(4 × 5 = 20)

- 1) Find Fourier series of $f(x) = x^2; -\pi \leq x \leq \pi$ and hence deduce

$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$$

- 2) Obtain the Fourier Series for $f(x) = \begin{cases} -K & ; -\pi \leq x \leq 0 \\ K & ; 0 \leq x \leq \pi \end{cases}$
- 3) Obtain sine half range Fourier series $f(x) = \begin{cases} \frac{l}{4} - x & ; 0 < x < \frac{l}{2} \\ x - \frac{3}{4} & ; \frac{l}{2} < x < l \end{cases}$
- 4) Find Fourier transform of $f(x) = \begin{cases} e^{imx} & ; a < x < b \\ 0 & ; \text{otherwise} \end{cases}$
- 5) Find sine transform of $f(x) = \begin{cases} x & ; 0 < x < 1 \\ 2 - x & ; 1 < x < 2 \\ 0 & ; \text{otherwise} \end{cases}$
- 6) Solve $\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial x^2}$ if $y(0, t) = 0, y(x, 0) = e^{-x}$.

