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SIVS-N192 A-17

B.A./B.Sc. IVth Semester Degree Examination

Mathematics

(Linear Algebra and Laplace Transform)

Paper : 4.1

(New)

Time : 3 Hours Maximum Marks : 60

Instructions to Candidates :

Answer all Sections.

SECTION - A

I. Answer any **ten** of the following : **(10 × 2 = 20)**

1) In a vector space V over the field F. Show that $C.\alpha = 0 \Rightarrow C = 0$ or $\alpha = 0$.

2) Prove that the intersection of any two subspaces of a vector space V over a field F, is also a subspace of V.

3) Find the subspace spanned by the set $S = \{(2, 0, 0), (0, 0, -2)\}$ in the vector space $V_3(R)$.

4) Show that the set $S = \{(1, 1, 1) (2, 2, 0), (3, 0, 0)\}$ is linearly independent.

5) The set of $(n-1)$ elements in an n-dimensional space V(F) cannot span V.

6) Define Basis and Dimension.

7) Find the Laplace Transform of $[e^{-4t} + 3e^{-3t}]$.

8) Find the Laplace Transform of $[\sin 3t.\cos 4t]$.

9) Find the Laplace transform of $e^{2t}(2t^2 - 3t + 4)$.

10) Find the Inverse Laplace Transform of $\frac{2S^2 - 5S + 8}{S^3}$.

11) Find the Inverse Laplace Transform of $\frac{1}{S^2 - 49}$.

12) State convolution theorem.

SECTION - B

II Answer any **four** of the following : **(4× 5 = 20)**

1) A non-empty subset W of a vector space V over F is a subspace of V if and only if

   i) $\forall w_1, w_2 \in W \Rightarrow w_1 - w_2 \in W$

   ii) $\forall C_1 \in F$ and $w_1 \in W \Rightarrow C_1 w_1 \in W$

2) Let V be a vector space over the field F, S and T be two non-empty subsets of V. Then

   i) $S \subseteq T \Rightarrow L[S] \subseteq L[T]$

   ii) $L[S] = S \Leftrightarrow$ S is a subspace of V.

3) Find the dimension and basis of the subspace spanned by the vectors (2, 4, 2), (1, –1, 0), (1, 2, 1) and (0, 3, 1) in $V_3(R)$.

4) Define $T : V_3(R) \rightarrow V_3(R)$ by $T(x_1, x_2, x_3) = (0, x_2, x_3)$ show that, T is a linear transformation.

5) Given the matrix $A = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 1 & 0 \end{pmatrix}$ determine the linear transformation $T : V_3(R) \rightarrow V_2(R)$ relative to bases B_1 and B_2 given by

   i) B_1 and B_2 are the standard bases of $V_3(R)$ and $V_2(R)$ respectively.

   ii) $B_1 = \{(1, 1, 1), (1, 2, 3), (1, 0, 0)\}$, $B_2 = \{(1, 1), (1, -1)\}$

6) Find the Range space, Kernel, Rank and Nullity of the linear transformation $T : V_2(R) \rightarrow V_2(R)$ defined by $T(x, y) = (x + y, x)$. Also verify the Rank-Nullity theorem.

SECTION - C

III Answer any **four** of the following. **(4 × 5 = 20)**

1) Find the Laplace transform of the Half curve rectifier

$$f(t) = \begin{cases} \sin(wt) & \text{for } 0 < t < \pi/w \\ 0 & \text{for } \dfrac{\pi}{w} < t < \dfrac{2\pi}{w} \end{cases}$$

2) Find the Laplace Transform of $\left(\frac{\cos at - \cos bt}{t}\right)$.

3) Express $f(t)=\begin{cases} t, & 0<t\leq 2 \\ t^2, & t>2 \end{cases}$ in terms of unit step function and hence find its Laplace Transform.

4) Find the Inverse Laplace transform of $\left(\frac{S}{(S-3)(S^2+4)}\right)$.

5) Find $L^{-1}\left[\frac{1}{(S^2+a^2)^2}\right]$ by using convolution theorem.

6) Solve $y''-9y=-8e^t$ given $y(0)=0,\ y'(0)=10$.