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SIIIS-N-195 B-17

B.Sc./B.A. IIIrd Semester Degree Examination

MATHEMATICS

(Groups, Rings, Line and Multiple Integral)

Paper - 3.1

(New)

Time : 3 Hours **Maximum Marks : 60**

Instructions to Candidates:

Answer all the questions.

SECTION - A

I. Answer any TEN of the following: **(10 × 2 = 20)**

1) Show that the multiplicative group of cube roots of unity is cyclic.

2) Find the orders of all the elements of an additive group of integer module 6 ie $(z_6\ ;\ +6)$.

3) Show that every cyclic group is abelian.

4) Find all the right cosets of the subgroup H = {0,3} in the group $(z_6;\ +6)$.

5) Define Homomorphism and Endomorphism.

6) Show that the mapping $f:(\mathrm{R},+)\rightarrow(\mathrm{R}^{+},\cdot)$ defined by $f(x)=e^{x}\ \forall x\in\mathrm{R}$ is a homomorphism.

7) Write the closure law, Associative law and distributive laws under Multiplication Operation of a Ring.

8) Define cancellation laws of Ring.

9) Evaluate $\int_C (2y+x^2)dx+(3x-y)dy$ along the, curve $x=2t, y=t^2+3$ where $0 \leq t \leq 1$.

10) Evaluate $\int_C (x+y)dx+(y-x)dy$ along the parabola $y^2=x$ from (1,1) to (4,2).

11) Show that $\int_0^1 \int_0^2 (x+y)\,dy\,dx = 3$.

12) Evaluate $\int_0^3 \int_1^2 (x^2+3y^2)\,dy\,dx$.

SECTION - B

II. Answer any **FIVE** of the following: **(5 × 5 = 25)**

1) Prove that Intersection of two subgroups is also a subgroup.

2) The order of an element 'a' of a group is the same as that of its inverse $a^{-1} i.e \cdot 0(a) = 0(a^{-1})$.

3) Let G be a finite group and H be any subgroup of G then 0(H)/0(G).

4) Prove that the set G/H of all cosets of a normal subgroup H of G group G is a group under the Binary operation defined by $H_a H_b = H_{ab}\ \forall H_a, H_b \in G/H$.

5) If f is a homomorphism of G into $\overline{G}$ then

   i) $f(e) = e^1$ where e and e^1 are Identities of G and G'.

   ii) $f(x^{-1}) = [f(x)]^{-1}\ \forall x \in G$.

6) If $G = \left\{ \begin{bmatrix} a & o \\ o & o \end{bmatrix} \middle| a \in R^* \right\rangle$, where R* = R – (0) Then show that $f:(G,\cdot) \to R^*$ defined by $f\begin{pmatrix} a & o \\ o & o \end{pmatrix} = a \forall \begin{bmatrix} a & o \\ o & o \end{bmatrix} \in G$ is an Isomorphism.

7) A ring without zero divisors if and only if cancellation laws holds in it.

8) The Necessary and sufficient condition for a non empty subset S of a ring R to be a subring are

i) $a \in s, b \in s \Rightarrow a - b \in s$

ii) $a \in s, b \in s \Rightarrow a \cdot b \in s$

SECTION - C

III. Answer any **THREE** of the following: **(3 × 5 = 15)**

1) Evaluate $\int_C (x^2 - 2xy)\,dx + (x^2 y - 3)\,dy$. Where C is the boundary of the region defined by $y^2 = 8x$ and $x = 2$.

2) Evaluate $\int_C (x + y + z)\,ds$ where C is the line joining the points (1,2,3) and (4,5,6).

3) Evaluate $\iint_R x^2 y^2\,dx\,dy$ where R is the circle $x^2 + y^2 = 1$.

4) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\,dx\,dy\,dz$.

5) Show that $\int_0^1 \int_0^1 \int_0^1 e^{x+y+z}\,dx\,dy\,dz = (e-1)^3$.

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